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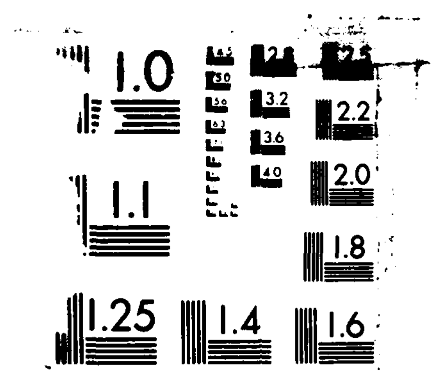
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LINEAR TIME INVARIANT SYSTEM IDENTIFICATION USING A RESULT OF THE BUSSGANG THEOREM

THESIS

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Second Lieutenant, USAF

AFIT/GE/ENG/86D-49

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THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science in Electrical Engineering

Timothy H. Lewis, B.S.E.E.

Second Lieutenant, USAF

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Tim Lewis

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List of Symbols

Symbol	Definition
$d'(t)$ . . .	The unit impulse function
$h(t)$ . . .	The impulse response of the LTI system
$H(f)$ . . .	The transfer function of a filter
$x(t)$ . . .	A random process or a sample function of the random process
X	The random variable obtained by sampling the random process $x(t)$ at some fixed t
$\overline{x}$	The mean value of the random variable X
s^2_x	The variance of the random variable X
$R_x(t)$ . . .	The autocorrelation of X
$S_x(f)$ . . .	The power spectral density of X
$p_X(.)$ . . .	The probability density function of X
$p_{XY}(.)$ . .	The joint density of X and Y
r	The correlation function of X and Y
$R_{xy}(t)$ . .	The crosscorrelation of X and Y
$S_{xy}(f)$ . .	The cross power spectral density of X and Y
$R_{yx'}(t)$ . .	The crosscorrelation of Y and the delayed version of X
$R_{yU}(t)$ . .	The crosscorrelation of Y and the delayed, hard limited version of X
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$R_{xx'}(t)$ . .	The crosscorrelation of X and the delayed version of X

Abstract

This investigation applied the Bussgang theorem to the cross-correlation method of linear, time invariant (LTI) system identification. In this procedure a Gaussian signal is passed through an LTI system and the output is crosscorrelated with a non-linearly distorted version of the original Gaussian signal. If the Gaussian noise were white the crosscorrelation would be equal to the impulse response of the LTI system within a constant of proportionality. With the use of bandlimited Gaussian noise this relationship is only approximately satisfied.

The analysis compared the performance of the crosscorrelation method with the non-linearity to that of the crosscorrelation method without the non-linearity. The experimental results indicate that the introduction of the non-linearity degrades the performance of the method, but the this can be improved by correcting for the effects of several quantities associated with the time varying statistics of the Gaussian noise.

LINEAR TIME INVARIANT SYSTEM IDENTIFICATION USING A RESULT OF THE BUSSGANG THEOREM

Chapter 1. Introduction

Background

A system H[.] is said to be linear, time-invariant (LTI) if and only if

$$y_1(t) = H[x_1(t)] \quad \text{and} \quad y_2(t) = H[x_2(t)] \tag{1.1}$$

implies

$$y_3(t) = H[x_3(t) = Ay_1(t-t_1) + By_2(t-t_2)] =$$

$$Ay_1(t-t_1) + By_2(t-t_2) \tag{1.2}$$

where $x_1(t)$ and $x_2(t)$ are arbitrary input signals, $y_1(t)$ and $y_2(t)$ are the corresponding outputs of the system, and t_1 and t_2 are arbitrary delays. Many systems encountered in electrical engineering satisfy the LTI condition, at least over a limited range of inputs, and techniques of LTI system identification (finding a mathematical description of the LTI system) are thus important to both theoretical work and practical applications.

The impulse response of a system, h(t), is the system's output when the input is a "spike" of infinite amplitude and zero time duration (an impulse). An LTI system is completely characterized by its impulse response. That is, if the impulse

response can be found, any characteristic of the system can then be derived from the impulse response. Thus the problem of identifying an LTI system is reduced to the problem of finding its impulse response.

One well known method of determining the impulse response of an LTI system is the crosscorrelation method, illustrated in Figure 1. In this method a white Gaussian (normally distributed) signal is the input to an LTI system. The crosscorrelation of the system output with the original noise, $R_{yx}(t_0)$, is proportional to $h(t_0)$, the LTI system's impulse response at time t_0. If the noise is ergodic (as is assumed in Figure 1), the crosscorrelation may be found by taking the average of the product of the delayed noise and the filter output. Bussgang (2:12) has shown that if the delayed noise is subjected to a non-linear device, the output of the crosscorrelation method will change only by a constant of proportionality (Figure 2).

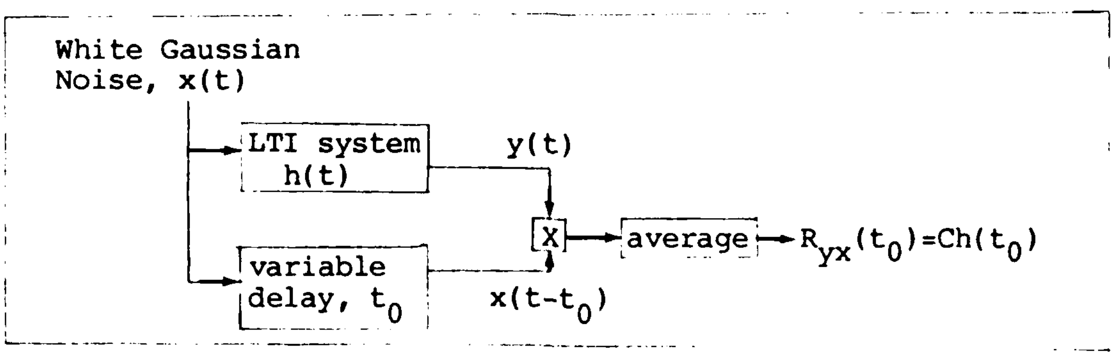

Figure 1. Crosscorrelation Method of LTI System Identification

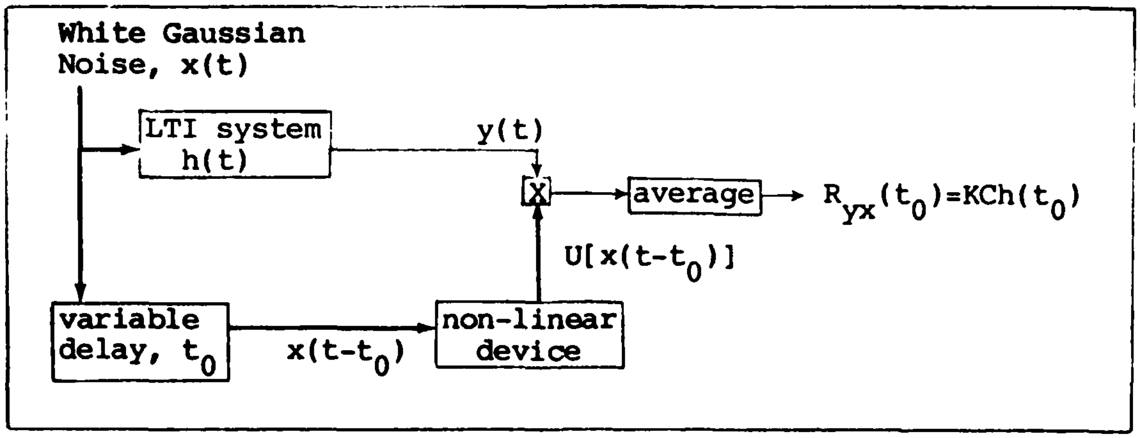

Figure 2. Modified Crosscorrelation
LTI System Identification Procedure

Problem and Scope

The purpose of this research effort is to implement the LTI identification procedure of Figure 2 and determine its ability to identify an LTI system. The non-linear device is a hard limiter defined by

$$U[x(t)] = \begin{cases} 1, & x(t) > 0 \\ 0, & x(t) \leq 0 \end{cases} \tag{1.3}$$

The LTI system identification procedure will be implemented on AFIT's SIMSTAR hybrid digital/analog computer. The delay function will be realized using an array to simulate a shift register. Two low pass LTI filters will be identified using this procedure.

Sequence of Presentation

The sequence of presentation of the report is as follows; chapter 2 covers the LTI identification procedure theory; chapter 3 describes the equipment used in the project; chapter 4 describes the SIMSTAR program used to implement the procedure, as well as the data aquisition process; chapter 5 contains the data analysis; and chapter 6 contains the conclusions, discussion, and recommendations.

Chapter 2. Mathematical Fundamentals

Linear, Time Invariant System

Since the purpose of this research effort is to investigate a method of identifying linear, time invariant (LTI) systems, a brief review of the properties of an LTI system is appropriate. A linear system is a system which obeys the superposition property: If an input to the system, $x_1(t)$, causes an output of $y_1(t)$, and an input of $x_2(t)$ causes an output of $y_2(t)$, then an input of $Ax_1(t) + Bx_2(t)$ causes an output of $Ay_1(t) + By_2(t)$, where A and B are arbitrary constants. A time invariant system is a system which has the following property: If an input of $x(t)$ causes an output of $y(t)$, then an input of $x(t-t_0)$ will cause an output of $y(t-t_0)$, where t_0 is an arbitrary time delay. Thus, a system which is both linear and time invariant will display the property that an input of $Ax_1(t-t_0) + Bx_2(t-t_0)$ will cause an output of $Ay_1(t-t_0) + By_2(t-t_0)$.

The behavior of any LTI system can be completely described in terms of its impulse response, $h(t)$, through the relationship

$$y(t) = h(t) * x(t) = \int h(a)x(t-a)da \qquad (2.1)$$

where * denotes convolution, and $x(t)$ and $y(t)$ are the system input and output, respectively. Note that all limits of integrations are from negative infinity to positive infinity unless noted otherwise.

A case of particular interest is when $x(t) = d'(t)$, the unit impulse, where $d'(t)$ is defined by the relationships

$$\int d'(t)dt = 1 \tag{2.2}$$

and

$$d'(t) = 0, \ t \neq 0 \tag{2.3}$$

Then, if $x(t) = d'(t)$ and the system is LTI,

$$y(t) = d'(t)*h(t) = \int d'(a)h(t-a)da = h(t) \tag{2.4}$$

where $h(t)$ is the impulse response of the system.

An LTI system's impulse response is related to the system's transfer function, $H(f)$, through the well known Fourier relationships

$$H(f) = \int h(t)\exp(-j2\pi ft)dt \tag{2.5}$$

$$h(t) = \int H(f)\exp(j2\pi ft)df \tag{2.6}$$

Random Processes

Let $x(t)$ be a stationary Gaussian (normal) random process. For any fixed value of time, t_1, the random variable X is created by sampling $x(t)$. That is, $X=x(t_1)$. The probability density function (pdf) of the random process $x(t)$ for any fixed t is given by

$$p_X(x) = \frac{1}{(2\pi)^{1/2}s_x} \exp\left(-\frac{(x-\bar{x})^2}{2s_x^2}\right) \tag{2.7}$$

where x is a dummy variable, s_x^2 is the variance of X, and $\overline{x}$ is the mean of X. The cumulative distribution function of X, $F_x(x)$, is

$$F_x(x) = Pr(X \leq x) = \int^{x} p_X(a)\, da \tag{2.8}$$

where a is a dummy variable of integration. (For ease of notation, x(t) will be used to denote either the ensemble of sample functions x(t) or a particular sample function (5:350).)

The mean of x(t) is given by

$$\overline{x} = E\{x(t)\} = \int x p_X(x)\, dx \tag{2.9}$$

where E{.} denotes the expectation operator. The variance of X is given by

$$s_x^2 = E\{[x(t)-\bar{x}]^2\} = \int [a-\bar{x}]^2 p_X(a) da \tag{2.10}$$

The autocorrelation of X is given by

$$R_x(t_0) = E\{XY\} = \int ab p_{XY}(a,b;t_0)\, da db \tag{2.11}$$

where a and b are dummy variables of integration, Y is the random variable $x(t-t_0)$ for any fixed t, and $p_{XY}(a,b;t_0)$ is the joint density of the jointly distributed random variables X and Y at any fixed t. If X and Y are jointly Gaussian (normal), then by definition the joint density for X and Y, $p_{XY}(x,y)$ is given by

$$p_{XY}(xy) = \frac{1}{2\pi s_x s_y (1-r^2)}$$

$$\exp\left(-\frac{[(x-\bar{x})/s_x]^2 - 2r(x-\bar{x})(y-\bar{y})/s_x s_y + [(y-\bar{y})/s_y]^2}{2(1-r^2)}\right) \quad (2.12)$$

where s_x^2 and s_y^2 are the variances of X and Y, and r denotes the correlation coefficient given by

$$r = \left| \frac{E\{(X-\bar{X})(Y-\bar{Y})\}}{s_x s_y} \right| \quad (2.13)$$

Note that Equation (2.12) denotes the joint p.d.f. of <u>any</u> two jointly Gaussian random variables, and is not restricted to the case where X and Y are samples of the same Gaussian process taken at different times.

The Fourier transform of $R_x(t)$ is the power spectral density of x(t):

$$S_x(f) = \int R_x(t)\exp(-j2\pi ft)\ dt \quad (2.14)$$

and clearly

$$R_x(t) = \int S_x(f)\exp(j2\pi ft)\ df \quad (2.15)$$

A case of particular importance for this report is the case when $S_x(f)$ is a constant for all values of frequency. In this case x(t) is said to be "white." If x(t) is white with $S_x(f) = N_0$, then $R_x(t) = N_0 d'(t)$. The importance of this result will be

discussed in the section dealing with the crosscorrelation method of LTI system identification.

Now let $y(t)$ be, like $x(t)$, a stationary Gaussian random process. A delayed version of $y(t)$, $y(t-t_0)$, will also be a stationary Gaussian random process. The crosscorrelation of $x(t)$ and $y(t-t_0)$ for fixed t is given by

$$R_{xy}(t_0) = E\{x(t)y(t-t_0)\} = \iint ab p_{XY}(a,b,t_0)\, da db \qquad (2.16)$$

where $p(x,y,t_0)$ is the joint density of the random variables $x(t)$ and $y(t-t_0)$ (at any fixed t), and t_0 has been included to stress the time dependence of the joint probability density function. If, in addition to being stationary, the two processes are ergodic (The ensemble average of a function of the process is equal to the time average (5:360)) then the crosscorrelation given above is equal to the time average of $x(t)y(t-t_0)$:

$$E\{x(t)y(t-t_0)\} = \lim_{T \to \infty} 1/(2T) \int_{-T}^{T} x(t)y(t-t_0)dt \qquad (2.17)$$

The Fourier transform of $R_{xy}(t_0)$ is the cross power spectral density of $x(t)$ and $y(t)$, $S_{xy}(f)$.

Finally, if $x(t)$ is the input to an LTI system and $y(t)$ is the output, it can be shown that (see Appendix A)

$$S_y(f) = S_x(f)|H(f)|^2 \qquad (2.18)$$

Crosscorrelation Method of LTI System Identification

The crosscorrelation method of LTI system identification is depicted in Figure 3. The input to the system is ergodic white Gaussian noise, $x(t)$, with power spectral density $S_x(f) = N_0$, and $h(t)$ is the impulse response of the unknown LTI system which we are to identify. Let $y(t)$ be the output of the unknown LTI in response to the input $x(t)$. Then $y(t)$ is multiplied by a delayed version of $x(t)$, and the result is averaged over time. Since the input noise is ergodic, $y(t)$ and $x(t-t_0)$ will be also, and the time average of $x(t-t_0)y(t)$ will be numerically equal to the expected value of $x(t-t_0)y(t)$.

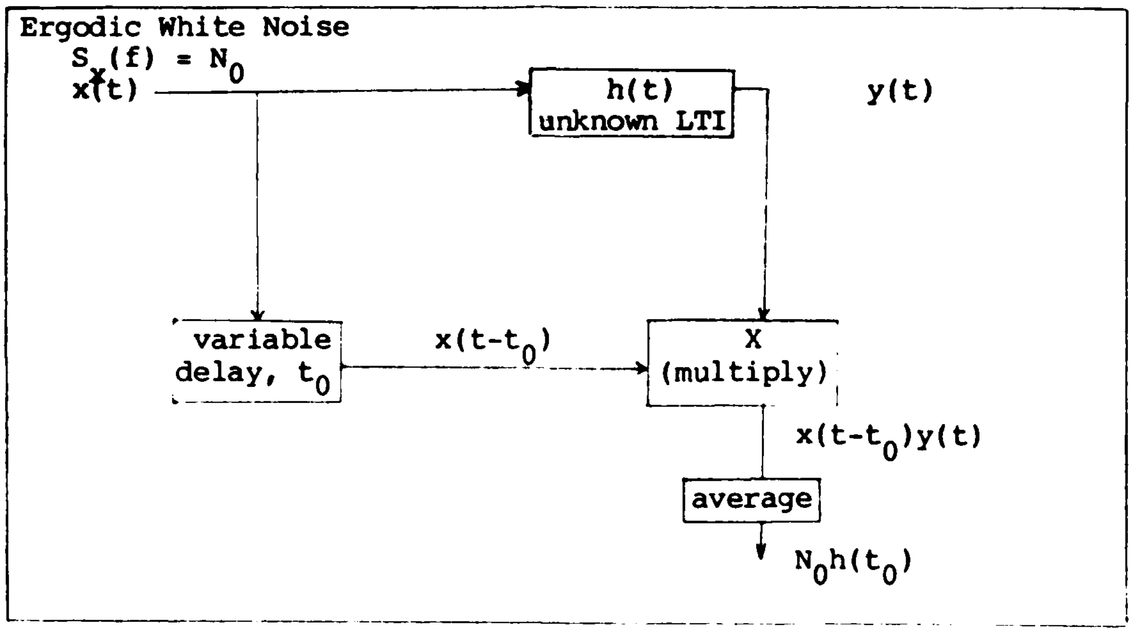

Figure 3. Crosscorrelation LTI Identification Technique

The assertion that the output of the averaging operation is $N_0h(t_0)$ is proven as follows:

$$E\{x(t-t_0)y(t)\} = E\{x(t-t_0)[x(t)*h(t)]\} =$$

$$E\{x(t-t_0) \int x(t-a)h(a)\ da\} = E\{\int x(t-t_0)x(t-a)h(a)\ da\} \quad (2.19)$$

Interchanging the order of integrations (recall that the expectation operator is essentially an integration over the sample space) results in

$$\int E\{x(t-t_0)x(t-a)h(a)\}\ da \quad (2.20)$$

Next, since h(a) is not a random variable, it may be moved outside of the expectation operator:

$$\int h(a)E\{x(t-t_0)x(t-a)\}\ da \quad (2.21)$$

Recalling the definition of autocorrelation, we have

$$\int h(a)R_x(t_0-a)\ da = h(t_0)*R_x(t_0) \quad (2.22)$$

Since x(t) was specified to be white noise with $S_x(f) = N_0$, it follows that $R_x(t_0) = N_0d'(t_0)$. Applying the sifting property of the delta function results in

$$h(t_0)*R_x(t_0) = h(t_0)*[N_0d'(t_0)] = N_0h(t_0) \quad (2.23)$$

so

$$E\{x(t-t_0)y(t)\} = N_0h(t_0) \quad (2.24)$$

and we have the desired result.

The Bussgang Theorem

A theorem due to Julian Bussgang (2:8-22) suggests a modification which will simplify the implementation of the cross-correlation method. The theorem states:

> "For two (zero mean) Gaussian signals, the cross correlation function taken after one of them has undergone nonlinear (instantaneous) amplitude distortion is identical, except for a factor of proportionality, to the crosscorrelation function taken before the distortion." (2:12)

A proof of the theorem for the case when both signals are zero mean follows:

Let y(t) and x(t) be jointly Gaussian random processes with zero means, and let U[x(t)] be a nonlinear function of x(t). Then $R_{Uy}(t)$ is the crosscorrelation of y(t) and the distorted version of x(t), U[x(t)], and is given by

$$R_{yU}(t_0) = E\{y(t)U[x(t-t_0)]\} =$$

$$\iint \frac{U(x)\ y}{2\pi s_x s_y (1-r^2)^{1/2}} \exp\left(-\ \frac{(x/s_x)^2 - 2rxy/s_x s_y + (y/s_y)^2}{2(1-r^2)}\right) dxdy \qquad (2.25)$$

To simplify the above expression, note that

$$\int y \exp\left(-\ \frac{(y/s_y)^2 - 2rxy/s_x s_y}{2(1-r^2)}\right) dy =$$

$$\int y \exp\left(-\ \frac{(y - rxs_y/s_x)^2}{2(1-r^2)s_y^{\,2}}\right) dy \exp\left(\frac{(rx)^2}{2(1-r^2)s_x^{\,2}}\right) =$$

$$\frac{rxs_y^2}{s_x} \; [2 \; \pi \; (1-r^2)]^{1/2} \; \exp \; \frac{r^2x^2}{s_x^2 2(1-r^2)} \tag{2.26}$$

Using this result, Equation (2.25) simplifies to

$$R_{yU}(t_0) = \frac{rs_y}{s_x^2(2\pi)^{1/2}} \int xU(x) \exp\left(-\frac{x^2}{2s_x^2}\right) dx \tag{2.27}$$

If we now define

$$K_v = \frac{1}{s_x^3(2\pi)^{1/2}} \int xU(x) \exp\left(-\frac{x^2}{2s_x^2}\right) \quad dx \tag{2.28}$$

(assuming the integral exists) it is apparent that

$$R_{yU}(t_0) = K_v r s_x s_y \tag{2.29}$$

$$\text{But } r = \frac{E\{y(t)x(t-t_0)\}}{s_x s_y} = \frac{R_{yx}(t_0)}{s_x s_y} \tag{2.30}$$

for any fixed t, so

$$R_{yU}(t_0) = R_{yx}(t_0) K_v \tag{2.31}$$

and we have the desired result.

Of particular interest is the case where $U[x(t)] = 1$ for $x>0$ and 0 for $x\leq 0$. In other words, the nonlinear device is a hard limiter, and its characteristic, U[.], is just an ideal step function of the argument. For this case

$$K_v = \frac{1}{(2\pi)^{1/2} s_x} \tag{2.32}$$

This result of the Bussgang theorem allows the crosscorrelation system identification technique to be implemented as shown in Figure 4.

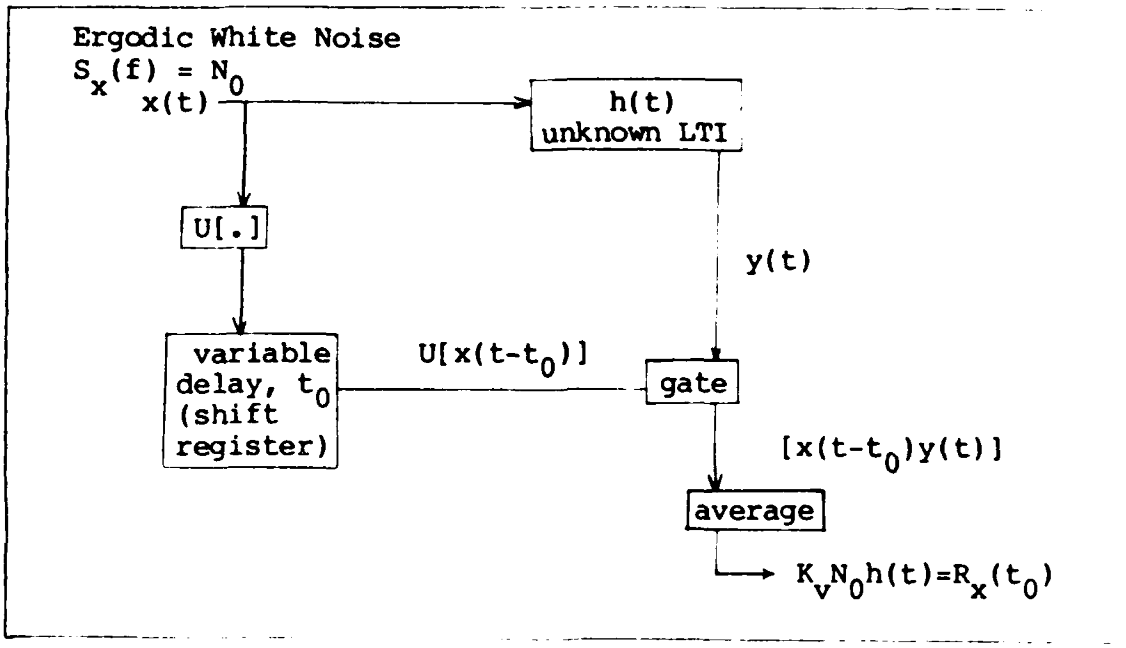

Figure 4. Modified Crosscorrelation LTI Identification Technique

Since U[x(t)] is either a one or a zero, the delay function can be implemented using a shift register. In addition, the multiplier of Figure 3 can be replaced with a gate which either passes or blocks y(t), depending on whether $U[x(t-t_0)]$ is one or zero. In other words if $U[x(t-t_0)]$ is one, the gate will allow y(t) to feed directly into the averaging operation, and if $U[x(t-t_0)]$ is zero, a zero will be fed into the averaging opera-

tion. Thus the Bussgang theorem allows a gate to replace the multiplier required in the correlator method of Figure 3.

System Limitations

The frequency response of any realizable system is limited. As a result, the "white" noise input, x(t), will in reality be bandlimited. In addition, the SIMSTAR computer on which the system identification process will be implemented has a finite frequency response, and will limit the bandwidth of the noise. These variations from the ideal case will now be analyzed.

In this project, the LTI systems which are to be identified both have passbands which cut off at frequencies well below the cutoff frequency of the input noise. Thus the bandlimited nature of the input noise will have virtually no effect on y(t), since frequencies outside its passband would be greatly attenuated even if the noise source were white. The power spectral density of the noise source used in this project is approximately flat over the range 0-1000 Hz. In order to have a precise mathematical model of the power spectral density of the noise, the noise is passed through a 2nd order Butterworth filter at the beginning of the LTI identification procedure. The transfer function of the Butterworth filter is

$$H_b(f) = \frac{(2\pi f_s)^2}{(j2\pi f + 2^{1/2}\pi f_s)^2 + (2^{1/2}\pi f_s)^2} \tag{2.33}$$

and its impulse response is

$$h_b(t) = 8^{1/2} \pi f_s \exp(-2^{1/2} \pi f_s t) \sin(2^{1/2} \pi f_s t)\, u(t) \quad (2.34)$$

where u(t) denotes a unit step function beginning at t = 0, and f_s = 500. The effects of this filter on the operation of the LTI identification procedure will be discussed shortly.

The use of a shift register to accomplish the delay function requires that $x(t-t_0)$ be a discrete time signal, $x'(t-t_0)$. This signal is simply a delayed version of the discrete signal x'(nT), where T is the sampling frequency of the A/D converter used to obtain x'(nT), and n is an integer index (n=1 indicates the first sample, n=2 indicates the second sample, etc.). Using a discrete time version of $x(t-t_0)$ will, in effect, bandlimit the input noise (this will be shown later). The result will be that $R_{xx'}(t_0)$, the crosscorrelation of x(t) with the discrete time signal, x'(t), will replace $R_{xx'}(t_0)$ in Figure 4. Instead of being a d' function, $R_{xx'}(t_0)$ will be a pulse of some finite time duration. Therefore, $R_{xx'}(t_0)*h(t_0)$ will not exactly equal $h(t_0)$. The width of $R_{xx'}(t_0)$ compared to the bandwidth of h(t) determines how closely $R_{xx'}(t_0)*h(t_0)$ approximates $h(t_0)$.

The LTI identification system at this point is depicted in Figure 5.

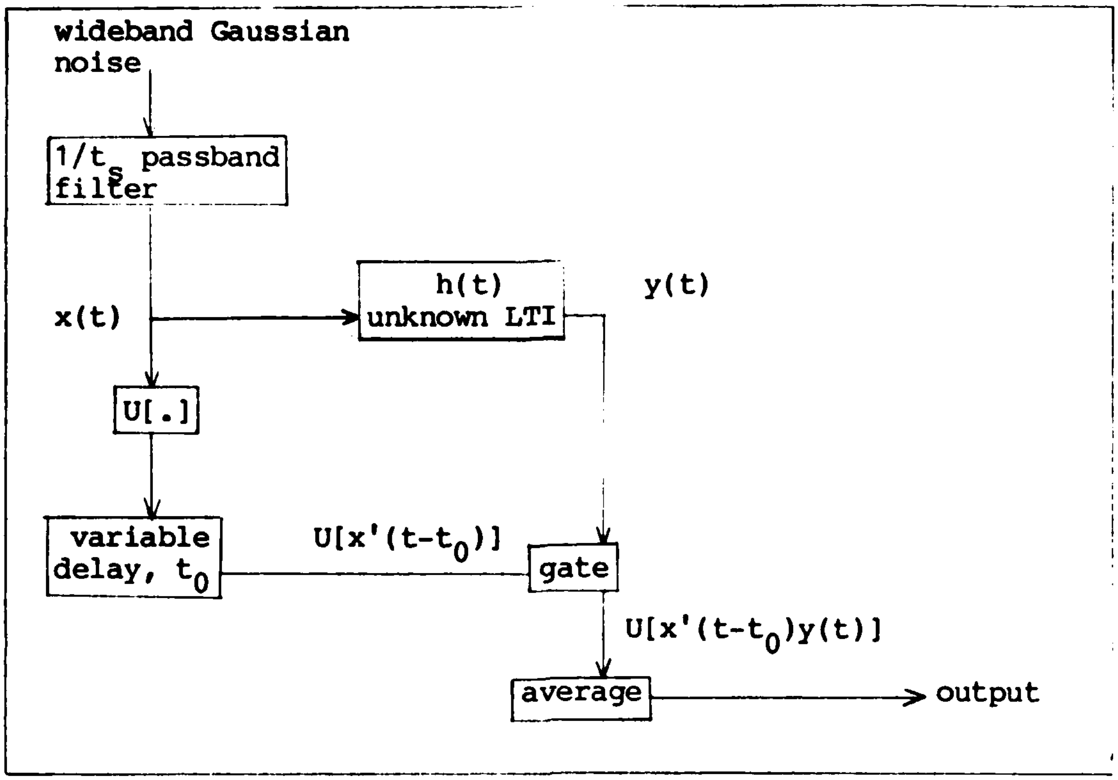

Figure 5. LTI Identification System With Components Replaced With Transfer Functions

At this point we must examine the effects of the introduction of the shift register and the Butterworth filter on the output of the system. Since the input Gaussian noise is essential& white within the passband of the Butterworth filter, the power spectral density of $x(t)$ is given by

$$S_x(f) = \frac{N_0}{1 + (f/f_s)4} \tag{2.35}$$

Taking the inverse Fourier transform gives

$$R_x(t) = \frac{N_0 \pi f_s}{2^{1/2}} \exp(-2^{1/2} \pi f_s |t|)[\cos(\sqrt{2}\pi f_s t) \pm \sin(\sqrt{2}\pi f_s t)] \quad (2.36)$$

where the + sign applies for $t>0$ and the - sign applies for $t<0$. The crosscorrelation of $x(t)$ and the delayed, discrete version of $x(t)$, $x'(t-t_0)$, will be found next.

The crosscorrelation of $x(t)$ and $x'(t)$ (invoking ergodicity) is

$$\begin{aligned} R_{xx'}(t_0) &= \langle x(t)x'(t-t_0)\rangle = \langle x(t+t_0)x'(t)\rangle \\ &= \langle x(nT+t'+t_0)x'(nT+t')\rangle \end{aligned} \quad (2.37)$$

where $\langle . \rangle$ denotes a time average and T denotes the sampling period of the A/D used to obtain $x'(nT)$. Since $x'(t-t_0)$ is a discrete time signal, $R_{xx'}(t_0)$ can only be found for values of t_0 which are integral multiples of T. If t' is restricted to the range $0<t'<T$ then we are assured that $x'(nT+t')$ is a constant and equal to $x(nT)$ (within the error limits of the quantizer used to sample $x(t)$). This is true because $x'(nT+t')$ with $0<t'<T$ is simply the value of a single sample of $x(t)$. Applying this result we have

$$\begin{aligned} R_{xx'}(t_0) &= \lim_{q\to\infty} 1/q \sum_{q/2}^{q/2} \{1/T \int_0^T x'(nT)x(nT+t_0+t')\, dt'\} \\ &= E\ \{1/T \int_0^T x'(nT)x(nT+t_0+t')\, dt'\} \end{aligned} \quad (2.38)$$

but $x'(nT)=x(nT)$ under the conditions imposed, so

$$= 1/T \int R_x(t_0+t')\,dt' \tag{2.39}$$

Carrying out the integration gives

$$R_{xx'}(t_0) = \begin{cases} (N_0 f_s/2)\{\exp(-\sqrt{2}\,\pi f_s t_0)\cos(\sqrt{2}\,\pi f_s t_0) \\ -\exp[-\sqrt{2}\,\pi f_s(t_0+T)]\cos[\sqrt{2}\,\pi f_s(t_0+T]\},\ t_0>0 \\ \\ (N_0 f_s/2)\{2-\exp[\sqrt{2}\,\Pi f_s(t_0+T)] \\ \cos[\sqrt{2}\,\Pi f_s(t_0+T)] \\ -\exp(\sqrt{2}\,\Pi f_s t_0)\cos(\sqrt{2}\,\Pi f_s t_0),\ -T<t_0<0 \\ \\ (N_0 f_s/2)\{-\exp(\sqrt{2}\,\Pi f_s t_0)\cos(\sqrt{2}\,\Pi f_s t_0) \\ +\exp[\sqrt{2}\,\Pi f_s(t_0+T)]\ \cos[\sqrt{2}\,\Pi f_s(t_0+T)],\ t_0<-T \end{cases} \tag{2.39}$$

A graph of $R_{xx'}(t_0)$ is shown in Figure 6. It should be noted that $R_{xx'}(t_0)$ is identical to the result obtained by passing the Butterworth filtered noise through a filter having a transfer function of $T\,\mathrm{sinc}(T\,f)\ \exp(-j\,\Pi t_s f)$. Thus, as mentioned earlier in this chapter, the result of using a discrete version of x(t) has the effect of low pass filtering the noise.

The output of the LTI identification procedure (see Fig 5) can now be found using the two-channel result (Appendix A).

$$R_{yx'}(t_0) = R_{xx'}(t_0) * h(t_0) \tag{2.40}$$

The effect of the U[.] device was omitted from the above equations. When the the U[.] device is included, the result is:

$$R_{yU}(t_0) = K_V R_{xx'}(t_0)*h(t_0) \tag{2.41}$$

In summary, the output of the LTI identification system will be h(t) convolved with $K_V R_{xx'}(t)$, as opposed to the ideal case, where h(t) is convolved with a delta function.

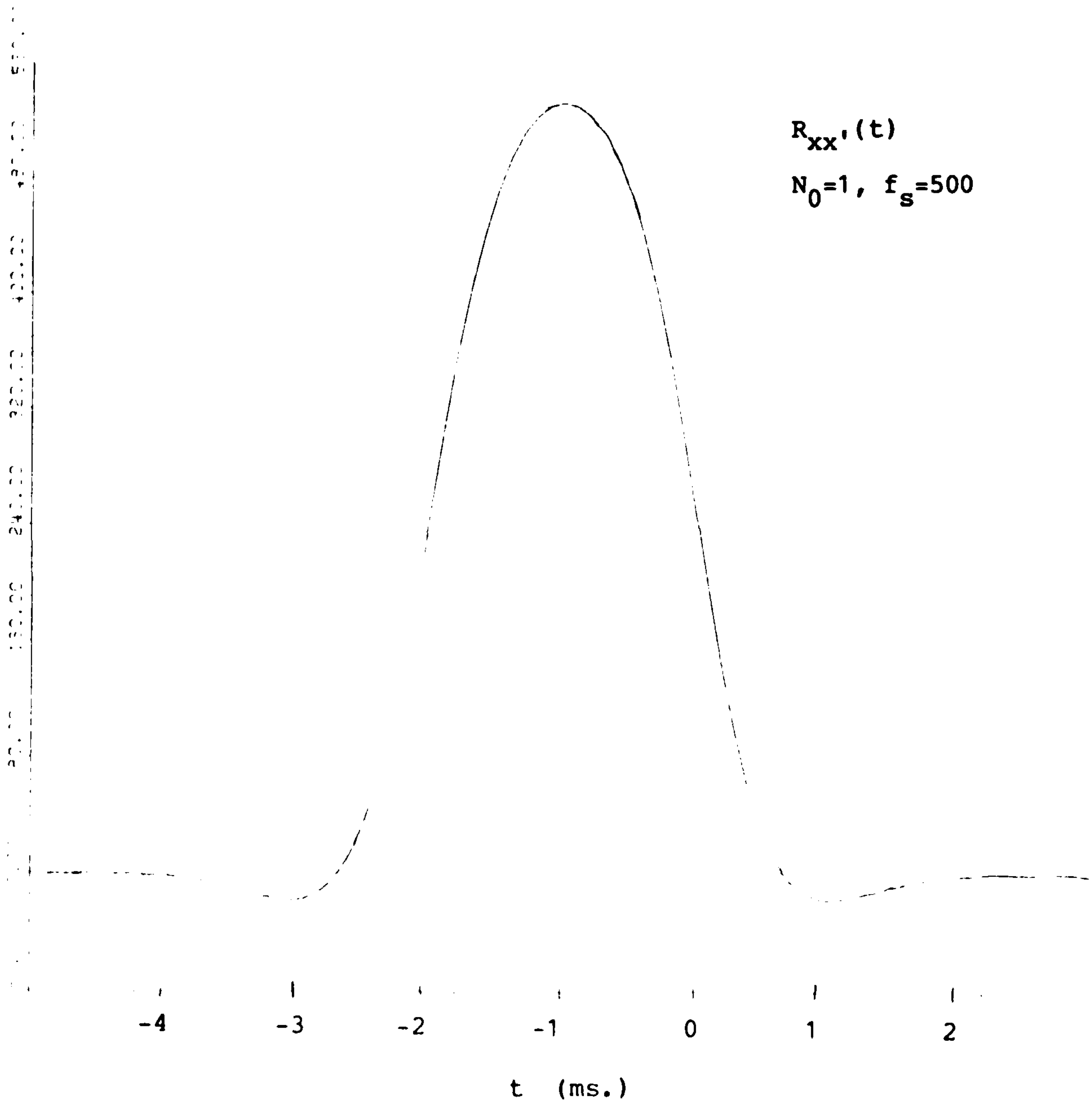

Figure 6. $R_{xx'}(t)$

CHAPTER 3. EQUIPMENT DESCRIPTION

SIMSTAR Hybrid Computer

The LTI identification procedure described in chapter 2 was implemented on an Electronic Associates, Inc. SIMSTAR hybrid (digital-analog) computer. The SIMSTAR consists of a Digital Arithmetic Processor (DAP), a Parallel Simulation Processor (PSP), an 8 channel strip chart recorder to record variables from the PSP, and a data logging option to store DAP and PSP variable values in a file for later analysis. The DAP is programmed using DTRAN, a high level Advanced Continuous Simulation Language (ACSL), or user supplied FORTRAN subroutines, which allows the user to implement additions, subtractions, multiplications, divisions, integrals, transfer functions, and special functions in a discrete-time simulation. The PSP is programmed using PTRAN (Parallel Translator, also an upgraded ACSL language), a continuous system simulation language which allows the user to model a system using high level statements to model integrals, derivatives, additions, subtractions, multiplications, transfer functions, logical functions, and special functions. The PSP and DAP can be programmed to exchange data using D/A and A/D converters at a conversion rate specified by the user.

To implement a simulation on the SIMSTAR the user writes the equations describing his simulation and implements some of the equations on the PSP and the rest on the DAP. The decision of

where to implement the equations is governed by the limited number of components (integrators, multipliers, etc.) available on the PSP. The equations with the highest frequency variables are implemented on the PSP until all available components are used, and the remaining equations are then assigned to the DAP. Care must be used when deciding which equations to implement on the DAP, since the relatively low communication rate between the DAP and the PSP (500 exchanges per second) will in general cause aliasing if the DAP variables have frequency components above 250 Hz.

Although the process of implementing a simulation on the SIMSTAR appears simple at first glance, it is complicated by the SIMSTAR's preliminary DTRAN and PTRAN compiler, which can best be described as user-hostile. Potential users are warned that the PSP "DEL" (delay) function is virtually unusable due to its poor frequency response, and the DAP "DEL" function has the sole effect of locking the SIMSTAR into an inoperable state which can only be cured by resetting the entire computer and reloading the operating system. Fortunately, the DAP supports user written FORTRAN programs, so features like "DEL" which don't work can be implemented with the user's own code. Caution is advised when using SIMSTAR's FORTRAN; however, because at least one function, AMOD (remainder), inexplicably multiplies the result by the divisor. To be safe, the user must carefully check the results of any FORTRAN or DTRAN/PTRAN statement.

White Noise Generator

The white noise generator used for this project was a Hewlet Packard 3722a Noise Generator set to 1.5 Khz, 3.16 volt rms, 20 volt peak-to-peak Gaussian noise.

CHAPTER 4. PROCEDURE

The Program

The LTI identification system described in chapter 2 was implemented on the SIMSTAR computer with the use of the programs Bus12 and Bus13. The two programs are identical except for the transfer function of the LTI to be identified. The Bus13 filter, filter A, is a low pass filter with the transfer function

$$H_A(f) = \frac{1}{(.0159j2\pi f) + 1} \tag{4.1}$$

and impulse response

$$h_A(t) = 62.832 \exp(-62.832t)\, u(t) \tag{4.2}$$

The transfer function of filter B, the Bus12 filter, is

$$H_B(f) = \frac{1}{3.567\ 10^{-3} f^2 + j11.459\ 10^{-3} f + 1} \tag{4.3}$$

and the impulse response is

$$h_B(t) = 105.68 \exp(-10t) \sin(104.72t)\, u(t) \tag{4.4}$$

A source code listing of Bus 13 is located at the end of this chapter (Figure 8). A general description of the program will be presented next. For detailed information concerning SIMSTAR programs the reader is referred to the PTRAN and DTRAN manuals.

The group of statements immediately following the PROGRAM

statement details the variable names used in the program, and the maximum and minimum values (scaling) of these variables. The SIMSTAR computer uses the scaling information to determine the settings of internal multipliers which limit all signals to plus or minus nine volts, the dynamic of the SIMSTAR's parallel processor. When a run is completed, SIMSTAR displays the value of any desired signal in terms of its original (unscaled) units. Note that DEL, the variable delay (in seconds) used in the LTI identification procedure is a PARAMETER. As such, it can be set before each run of the program without recompiling the program.

Also note that the constant MAXT = .002 sets the sampling period of the A/D and D/A converters to .002 seconds.

The DERIVATIVE section of the program contains both the digital and parallel (analog) instructions which define the LTI identification procedure. The first statement in the DERIVATIVE region invokes the subroutine DELAY1. DELAY1 stores the current value of X (the gaussian noise) and returns a delayed value of X, (XP), as well as BA, the hard limited version of XP. Note that the variables XP and BA had to be renamed XP1 and B in order to force the DERIVATIVE region to accept them.

The PARALLEL region, contained within the DERIVATIVE region, includes all the equations which are implemented in the analog domain. It should be noted that the order in which instructions are placed in the PARALLEL region is unimportant, because SIMSTAR executes all parallel instructions simultaneously. The output of the Gaussian noise generator, X1, is passed through the Butter-

worth filter. The result, X2, is added to X2BAR to overcome a slight (-.058 volt) bias to produce X, a nearly zero mean, band-limited Gaussian signal. This signal is passed through the LTI which is to be identified. The output of the LTI is Y. This signal is multiplied by B (the delayed, hard limited version of X) to produce C. An integrator produces E1, the integral of C. Finally, E1 is divided by T (time) to give E2, the time average of the LTI output multiplied by the hard limited, delayed Gaussian noise.

Several other averages which are computed are:

XBAR, the average of the bandlimited Gaussian noise,

CHECK, the average of the delayed Gaussian noise,

SIGXPR, the average of the square of the delayed Gaussian noise,

CHECKY, the average of the LTI output,

SIGMAY, the average of the square of the LTI output,

EE2, the crosscorrelation of the delayed (but not hard-limited) Gaussian noise with the LTI output.

Figure 7 presents the operation of the SIMSTAR program in graphic format. For clarity, integrators followed by a division by time are indicated simply as averagers.

Data Acquisition

Data for the LTI identification procedure was taken by conducting a series of one minute runs and recording the results. The value of DEL was specified before each run, and three runs were performed for each value of DEL. Because of the discrete

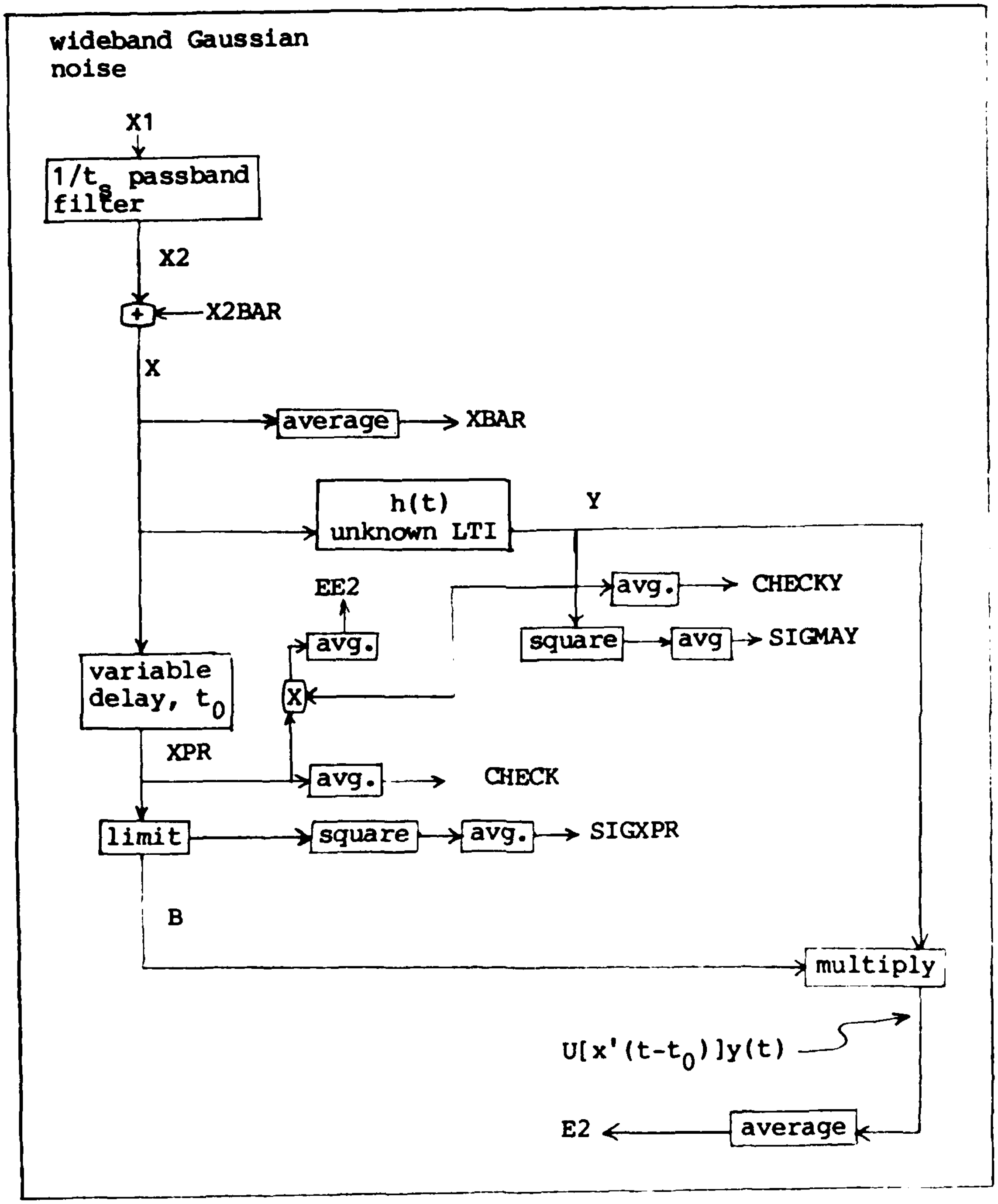

Figure 7. LTI Identification
System as Implemented on the SIMSTAR.

time nature of the delay function, the only values of DEL available were integer multiples of the sampling period of the system, .002 seconds. At the conclusion of each run the values of XBAR, CHECKY, CHECK, SIGXPR, SIGMAY, E2, and EE2 were recorded. Appendix C contains the raw data as well as graphs of the data.

```
*PSP=1,0,ERR=ALL
*TITLE
BUS13
*INPUT
PROGRAM
   CONSTANT TMAX=61 ,DEL=1.0,P=.015915491,CINT=1,MAXT=.002,...
   X2BAR = -.068,BMAX=1,XP1MAX=9 ,PA=90.36E-6,PB=1.8238E-3
   '@PARAMETER DEL,TMAX,P,CINT,MAXT,P2,P3,X2BAR'
   '@MAXVAL DEL = 1  , TMAX = 61 , T = 61 , P=2,CINT=10  ,...
    E1=6  ,     MAXT=.002,P2=228E-9,P3=675E-6,SIGX1=340 ,SIGY1=120 ,...
    SIGXPR=6 ,SIGMAY=2 ,EE1=30  '
   '@MINVAL DEL=.001 ,TMAX= 0, T =.1   ,P=.0001,CINT=.01,...
    E1= -6 ,     MAXT = .0015,P2=57E-9,P3=337E-6,SIGX1=0,SIGY1=0,...
    SIGXPR=0,SIGMAY=0,EE1=-30'
   '@SCALE X1=9, Y=9,X=9,STP=1, X2=9,C=9 ,E2=.10,B=BMAX,...
    MEA=6 , MEAN=.1,CHECK1=6 ,CHECK=.1,C1=9,EE2=.5,CHCKY1=6 ,...
    CHECKY=.1,XP1=9,XPR=9,XP=9,  XBAR1=30,XBAR=.5 '
   '@EXTERN X1'
   ' INTDEF(0,1,1) '
    'INTDEF(1,1,0)'
INITIAL
    NSTEPS NSTP = 1
    BA=0
     CALL SETUP(DEL,MAXT,K,I,RINT)
END $'OF INITIAL'
   DYNAMIC
    'INTERRUPT RATE ERROR DECLARATIONS'
         LOGICAL ENDER1, RATER1,ERROR1
         ENDER1 = .FALSE.
         ERROR1 = RATER1
         MINTERVAL MINT = 2.0E-6
         MAXTERVAL MAXT = .1
DERIVATIVE
         CALL DELAY1(X,BA,K,I,RINT,XP)
         INTEGER I,K
         B=BA
         XP1=XP
'@PARALLEL'
    XPR=XP1*STP
    DEL1 = DEL
    P3 = MAXT*.22508
    P2 = MAXT**2/39.478
    X2= CMPXPL(P2,P3,X1,0,0)
    MEA = INTEG(X2,0)
    MEAN = MEA/T
```

Figure 8. Simstar Program

```
      X = X2 - X2BAR
      CHECK1 = INTEG (XPR,0)
      CHECK = CHECK1/T
      CHCKY1=INTEG(Y,0)
      CHECKY =CHCKY1/T
      SIGX1 = INTEG(XPR**2,0)
      SIGXPR = SIGX1/T
      SIGY1 = INTEG(Y**2,0)
      SIGMAY = SIGY1/T
      Y = REALPL (P,X,0)
      STP=STEP(DEL)
     B1=  B*STP
     C = Y*B1
      E1 = INTEG(C,0)
      E2 = E1/T
      C1=Y*XPR
      EE1=INTEG(C1,0)
      EE2=EE1/T
      XBAR1 = INTEG(X,0)
      XBAR = XBAR1/T
      TERMT(T .GT. TMAX)
      '&RECORD(REC01,,,,,,,,,Y      ,EE1  ,EE2  ,XPR,E1,E2,SIGY1,CHECK)'
        GPI0 = CLOCK(MAXT)
        GPI1 = CLOCK(CINT)
        '@INTRRT 1 = GPI0'
        '@INTRRT 2 = GPI1'
         RATER1 = RATERR(GPI0,ENDER1)
    '@END PARALLEL'
   END $'OF DERIVATIVE '
  END   $'OF DYNAMIC'
 TERMINAL
 END $'OF TERMINAL'
 END $'OF PROGRAM'
 *TRANSLATE

 CONNECT AIN30 = X1
         DCA(1) = B
         DCA(2) = XP1
         PADC(1) = X
 *OUTPUT
 *END

         SUBROUTINE PREP1
 +
          INCLUDE E1.BUS13
          X = GRPADC(0)*S:X
          RETURN
          END
```

Figure 8 (continued).

```
C
      SUBROUTINE POST1
+
      INCLUDE E1.BUS13
      COMMON /QQDCP/DCASF(C:1)
      LOGICAL DELAY
      CALL QWDCAR(C,B*DCASF(0))
      CALL QWDCAR(1,XP1*DCASF(1))
      IF (L:RATER1) CALL ZZRTER(1)
      L:ENDER1 = .TRUE.
      DELAY = L:ENDER1
      L:ENDER1 = .FALSE.
      RETURN
      END
C
      SUBROUTINE PREPDCA
+
      COMMON /QQDCP/DCASF(0:1)
      DCASF(0) = 1.0/QDCASR(0)/BMAX
      DCASF(1) = 1.0/QDCASR(1)/XP1MAX
      RETURN
      END

      SUBROUTINE SETUP(DEL,DT,K,I,RINT)
      DIMENSION Y(1000)
      COMMON /ONE/ Y
      DO 10 I=1,1000
 10   Y(I) = 0.0
      A = DEL/DT
      K = AINT(A)
      RIN  = AMOD(DEL,DT)
      RINT = RIN/DT
      I = 0
      TYPE *,'K = ',K,' RINT = ',RINT,' I = ',I
      RETURN
      END

      SUBROUTINE DELAY1(X,XX,K,I,RINT,XP)
      DIMENSION Y(1000)
      COMMON /ONE/Y
      I = I+1
      IF (I .GT. 1000) I=I-1000
      Y(I) = X
      J = I-K
      IF (J .LT. 1) J = J+1000
      J1=J-1
      IF (J .EQ. 1) J1=1000
      XP = Y(J) + RINT*(Y(J1)-Y(J))
      IF (XP .GT. 0) XX=1
      IF (XP .LT. 0) XX=0
      RETURN
      END
```

CHAPTER 5. DATA ANALYSIS

Review

As previously described, the expected value of the product of a t_0-second delayed white noise process and the output of an LTI system with the same white noise input is simply the impulse response of the LTI convolved with a delta function (scaled by a multiplicative constant dependent on the power spectral density of the white noise). If the noise is not white, then the delta function is replaced by the autocorrelation of the noise. If the delayed noise is replaced by a discrete time version of the noise, the autocorrelation of the noise is replaced by the cross-correlation of the input noise with the discrete time version of the noise. If the noise source is zero mean Gaussian and the delayed noise is subjected to some instantaneous nonlinearity, the sole effect is to scale the output by a constant, K_V.

Non-Zero Mean Noise

For the case where the LTI identification procedure input noise is zero mean, it was shown in Chapter 2 that the output of the LTI identification procedure, R_{yU}, is given by

$$R_{yU}(t_0) = K_V R_{xx'}(t_0) * h(t_0) \qquad (2.41)$$

In the more general case where the input noise is not zero mean,

it can be shown (Appendix B) that

$$R_{yU}(t_0) = \frac{\{R_{yx'}(t_0)-\bar{x}\bar{y}\}\exp(-\bar{x}^2/2s_x^{\,2})}{\sqrt{2\Pi}\,s_x} + y/2\{1+\mathrm{erf}(x/\sqrt{2}s_x)\} \quad (5.1)$$

and so

$$R_{yx'}(t_0)=\{R_{yU}(t_0)-\bar{y}/2[1+\mathrm{erf}(\bar{x}/\sqrt{2}s_x)]\}\sqrt{2\Pi}\,s_x\ \exp(\bar{x}^2/2s_x^{\,2})+\bar{x}\bar{y} \quad (5.2)$$

Using this result, experimental values of $R_{yU}(t_0)$, $\bar{x}$, $\bar{y}$, s_x, and s_y can be combined to find an approximation of $R_{yx'}(t_0)$, the time average crosscorrelation of the input noise with the filter output. This approximation of $R_{yx'}(t_0)$ is labeled $R'_{yx'}(t_0)$ and its value for each run is included with the raw data in Appendix C.

Estimating The Impulse Response

The goal of the LTI identification procedure is, of course, to estimate h(t) by applying the Bussgang theorem to the crosscorrelation technique. Three crosscorrelations are of interest for comparing their effectiveness in estimating h(t):

1. $R_{yx'}(t_0)$, the crosscorrelation of the filter output with the delayed (but not hard limited) noise is the baseline.
2. $R_{yU}(t_0)$, the crosscorrelation of the filter output with the delayed, hard limited noise is related to $R_{yx'}(t_0)$ by Equation (5.2).
3. $R'_{yx'}(t_0)$, the result of applying Equation (5.2) to $R_{yU}(t_0)$,

is distinguished from $R_{yx'}(t_0)$ by the "'" to indicate that it is an approximation of $R_{yx'}(t_0)$.

In order to compare the effectiveness of these three crosscorrelations in estimating h(t), it is necessary to examine more closely the relationship between the crosscorrelations and the impulse response.

As was noted earlier

$$R_{yx'}(t_0) = R_{xx'}(t_0)*h(t_0) \tag{5.3}$$

If $R_{xx'}(t_0)$ is approximated by M d'(t+.001), an impulse of amplitude M located at t = -.001 (a good approximation as long as h(t) varies slowly with respect to the duration of $R_{xx'}(t_0)$), then the previous equation becomes

$$R_{yx'}(t_0) = M\ h(t + .001) \tag{5.4}$$

All that remains to be done to estimate h(t) from $R_{yx'}(t_0)$ is to determine M. One method of doing this would be to obtain experimental values of $R_{xx'}(t)$ (by setting H(f) = 1) and find the area under the curve of Equation (2.40). The problem with this method is that the only point on the $R_{xx'}(t)$ curve (see Figure 6) for which significant data may be taken is the t = 0 point. This is true because the only valid values of delay are positive integral multiples of .002 seconds (there is no way for the shift register to predict the value of x(t) in order to allow negative values of delay to be used), and $R_{xx'}(t)$ is near zero for delays of .002 seconds or more. Knowing only one point of $R_{xx'}(t)$ would

make it difficult to accurately determine the height of the curve. Thus it would be difficult to integrate the area under the curve.

A better method of determining M is to simply use the LTI identification technique on a known filter and fit the resulting data points to the known curve by changing M to achieve the least square error between the known impulse response and the data points. That is, find the least square error fit of $(1/M)R'_{yx'}(t)$ to $h(t+.001)$. The value of M thus obtained could then be used with the data from unknown filters to determine their impulse responses. Since $R'_{yx'}(t_0)$ is simply an approximation of $R_{yx'}(t_0)$, the same procedure may be applied to $R'_{yx'}(t_0)$ to estimate h(t).

The problem of estimating h(t) from the values of $R_{yU}(t_0)$ can be handled in a similar fashion. If the noise is approximately stationary from run to run (it was assumed to be stationary in the derivations, but stationarity applies to the equality of expectations and infinite time averages) then $\bar{x}$, $\bar{y}$, s_x, and s_y will be approximately constant from run to run. In this case (if $\bar{y}$ and $\bar{x}$ are small so that the additive $\bar{x}\bar{y}$ term can be ignored) Equation (5.2) simplifies to a multiplicative factor approximately constant from run to run. Then a least squares fit of $R_{yU}(t_0)$ to M' $h(t-.001)$ for a known filter will provide the desired constant needed to estimate an unknown h(t) from values of $R_y(t_0)$.

The values of $(1/M)R'_{yx'}(t)$, $(1/M)R_{yx'}(t)$, and $(1/M')R_{yU}(t_0)$ for filters A and B, scaled as just described, are plotted in Figures 8 and 9 along with the theoretical lines for $h_A(t)$ and $h_B(t)$ (The inset in Figure 8 is simply the data for filter A plotted on the same scale as Figure 9 for easy comparison). The values of M and M' required to obtain the least squares fit of $(1/M)R'_{yx'}(t)$, $(1/M)R_{yx'}(t)$, and $(1/M')R_{yU}(t_0)$ to h(t), along with the resulting mean square errors, are shown in Table 1.

Table 1. Scaling Factors and Mean Square Errors Resulting From Fitting Data to $h_A(t)$ and $h_B(t)$.

Filter	1/M for best fit of $R_{yx'}$	mean sq error	1/M for best fit of $R'_{yx'}$	mean sq. error	1/M' for best fit of R_{yU}	mean sq. error
A	318	1.38	318	2.66	1486	35.6
B	313	12.9	313	19.1	1425	46.1

It is clear that estimations based on $R_{yU}(t_0)$ data are far less accurate than estimates based on $R'_{yx'}(t_0)$. This is to be expected, since Equation (5.2) takes into account variations of $\bar{x}$, $\bar{y}$, s_x and s_y from run to run, while the least squares fit of $(1/M')R_{yU}(t_0)$ to h(t) ignores these run-to-run variations.

Comparison of the mean square error between estimates of h(t) based on $R_{yx'}(t_0)$ and those based on $R'_{yx'}(t_0)$ reveals that the latter are somewhat less accurate. This requires some explanation, since Equation (5.2) predicts that the two results

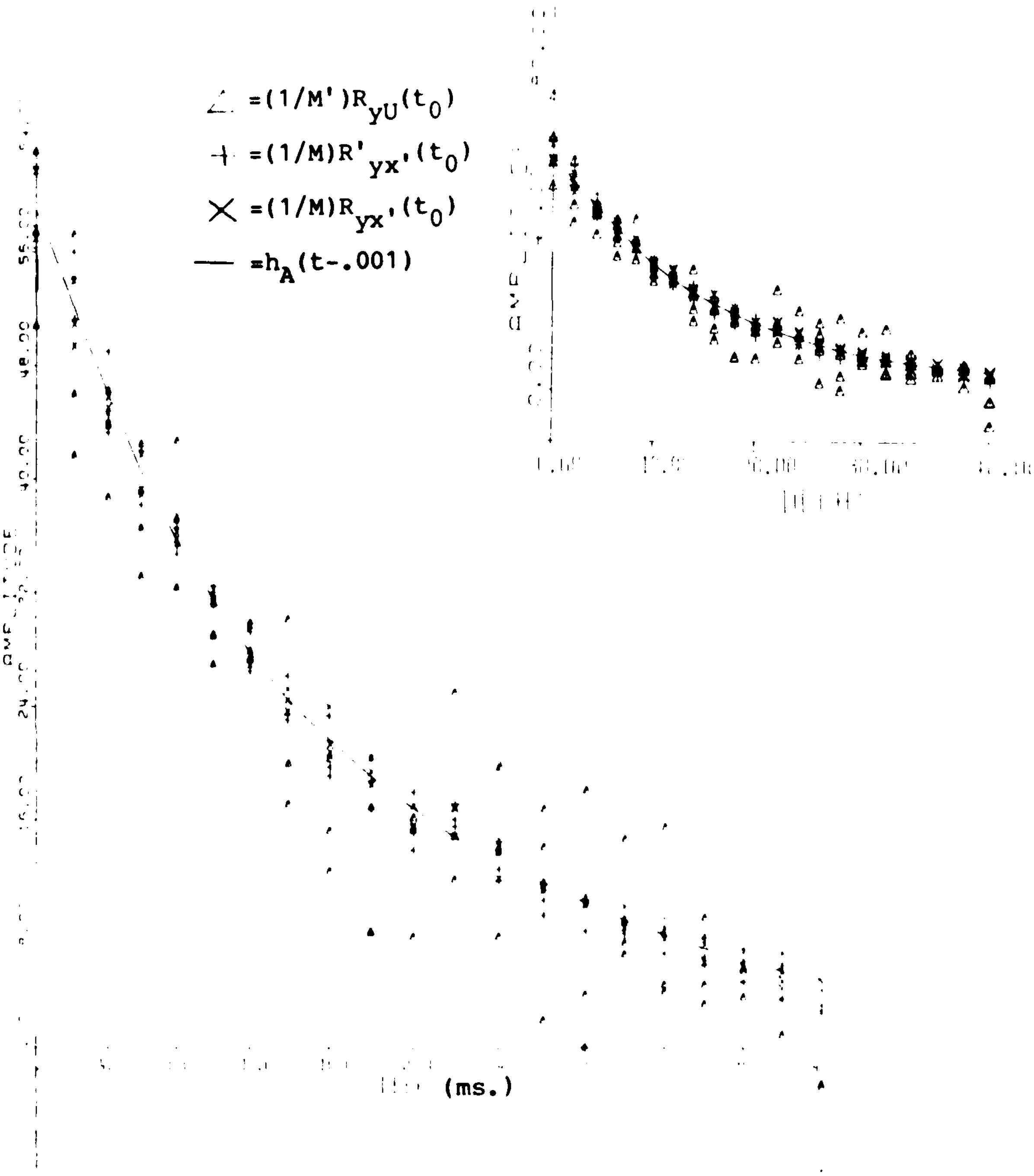

Figure 8. Filter "A" Graph

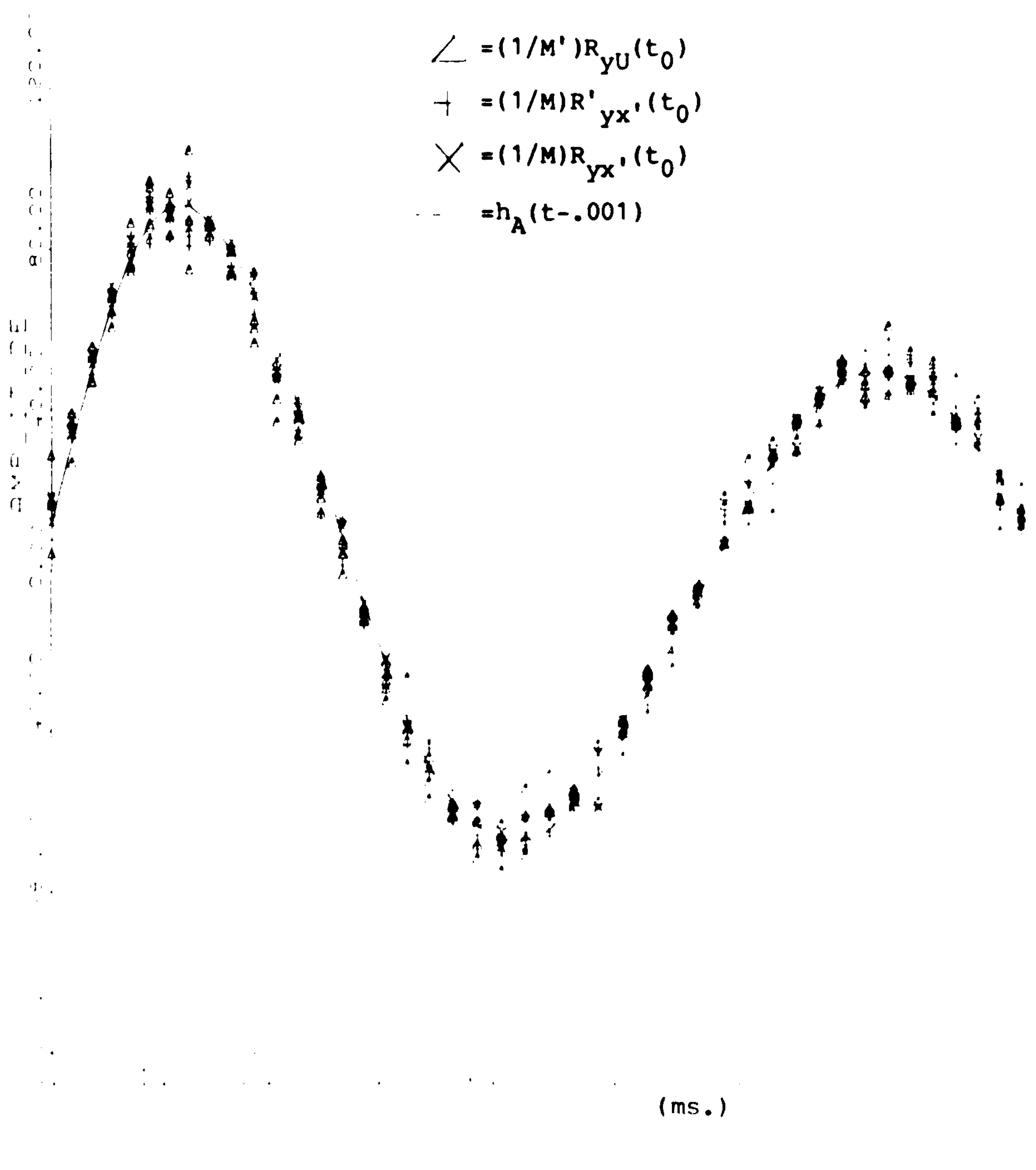

Figure 9. Filter "B" Graph

should be identical. A probable explanation lies in the assumption of ergodicity. The ergodic assumption is that the expectation value of a function of the random process is numerically equal to the corresponding time average (over all time) taken on any sample function of the process (5:360). The averages for this project were taken over a finite time interval, so some variation between particular values of $R_{yx'}$ and $R'_{yx'}$ is to be expected. In addition, the hard limiting accomplished by the U[.] device would be expected to erode the precision of fairly short-term time averages of functions of the hard limited signals. (For example, consider an infinitesimal time average of $x(t)U[x(t)]$. In general, Equation (5.2) will not be able to predict $x(t)x(t-t_0)$.)

As was mentioned earlier, estimations of $h(t)$ based on $R_{yx'}(t_0)$ are more accurate than those based on the other two crosscorrelations. The issue of errors between $(1/M)R_{yx'}(t_0)$ and $h(t_0)$ will now be addressed.

A cause of error throughout the project is that resulting from the finite (three orders of magnitude) dynamic range of the SIMSTAR, and the inevitable inaccuracies associated with analog components. In addition, the accuracy of the A/D-D/A converter combination is on the order of 5-6 mv (3). Scaling this by 1/M gives an A/D-D/A-based error of up to about 1.9 volts or a square error of about 3.5 volts^2. This is significant when compared to the mean square errors between $h(t_0)$ and $(1/M)R_{yx'}(t_0)$.

Chapter 6. Conclusions, Discussion, and Recommendations.

Conclusions

When Equation (5.2) is used, the LTI identification procedure produced results of the same order of accuracy as the traditional crosscorrelation method. When Equation (5.2) is not used the resulting approximation, $(1/M')R_{yU}(t_0)$, is significantly less accurate.

Discussion

The original impetus for this project was the idea that the use of the Bussgang result might allow the crosscorrelation method to be performed with less expensive equipment than would be required without it. The principal savings achieved by the "Bussgang" approach are that the delay function may be accomplished by a one-bit-word shift register instead of a full precision shift register, and the full precision A/D converter required by the conventional crosscorrelation approach may be replaced by a device which makes a simple 1 or 0 decision. The principal drawback to the "Bussgang" approach is that several integrators and squarers are required to determine $\overline{x}$, $\overline{y}$, s_x, and s_y in order to obtain accuracy close to that of the conventional crosscorrelation method.

Recommendations

The mean square error between $(1/M)R_{yx'}(t)$ and the theoretical impulse response was nearly an order of magnitude worse for filter B than for filter A. Testing of additional filters might provide insight into the source of this increased error. Also, it may be possible to achieve reasonable predictions of the impulse response of an LTI system by using only some of the terms of Equation (5.2). Further analysis could determine which terms could be omitted without seriously reducing the accuracy of the final results.

Finally, the effect of increasing the time of each run warrants further investigation. It is likely that longer runs would produce better results by reducing any errors caused by violating the infinite-time-average condition of ergodicity.

Appendix A. The Two Channel Result.

Prove: $R_{y1\ y2}(t') = R_{x1\ x2}(t')*h_1(t')*h_2(t')$

$$x_1(t) \longrightarrow \boxed{h_1(t)} \longrightarrow y_1(t)$$

$$x_2(t) \longrightarrow \boxed{h_2(t)} \longrightarrow y_2(t)$$

$$S_{y1\ y2}(f) = \quad E\{y_1(t)y_2(t-t')\}\exp(-j2\pi ft')\ dt'$$

$$= \int E\{\iint h_1(a)x_2(t-a)h_2(b)x_2(t-b-t')\ da\ db\ \exp(-j2\pi ft')\}dt'$$

$$= \iiint h_1(a)h_2(b)R_{x1\ x2}(b+t'-a)\ da\ db\ \exp(-j2\pi ft')\ dt'$$

$$= \iint h_1(a)h_2(b)S_{x1\ x2}(f)\ \exp[-j2\pi f(a-b)]\ da\ db$$

$$= H_1(f)H_2^*(f)S_{x1\ x2}(f) \qquad \text{(A.1)}$$

so

$$R_{y1\ y2}(t') = R_{x1\ x2}(t')*h_1(t)*h_2(-t) \qquad \text{(A.2)}$$

For the case at hand (Equation 2.40) $h_1(t')=h_y(t')$, $h_2(t')=d'(t')$, $R_{x1\ x2}(t')=R_{xx'}(t')$ and $R_{y1\ y2}(t')=R_{yx'}(t')$.

APPENDIX B. NON-ZERO MEAN BUSSGANG DERIVATION

This derivation is carried out for the special case of U[x(t)] being a hard limiter (Equation (1.1)).

$$R_{yU}(t_0) = E[y(t)U\{x(t-t_0)\}]$$

$$= \iint \frac{yU(x)}{2\Pi s_x s_y \ (1-r^2)^{1/2}} \cdot$$

$$\exp\left(- \frac{\{(x-\bar{x})/s_x\}^2 - 2r\{(x-\bar{x})/s_x\}\{(y-\bar{y})/s_y\} + \{(y-\bar{y})/s_y\}^2}{2(1-r^2)}\right) dxdy \qquad \text{(B.1)}$$

Note that

$$\int y \ \exp\left(- \frac{-2r\{(x-\bar{x})/s_x\}\{(y-\bar{y})/s_y\} + \{(y-\bar{y})/s_y\}^2}{2(1-r^2)}\right) dy$$

$$= \int y \ \exp\left(- \frac{[y-(\bar{y}+(rs_y/s_x)(x-\bar{x}))]^2}{2(1-r^2)s_y^{\ 2}}\right) dy \left(\exp \ \frac{r^2(x-\bar{x})^2}{2(1-r^2)s_x^{\ 2}}\right)$$

$$= (\bar{y} + rxs_y/s_x - r\bar{x}s_y/s_x) \ \exp\left(\frac{r^2(x-\bar{x})^2}{2(1-r^2)s_x^2}\right) [2\Pi(1-r^2)s_y^{\ 2}]^{1/2} \qquad \text{(B.2)}$$

so

$$R_{yU}(t_0) = \int \frac{U(x) \ \{\bar{y} + (rs_y/s_x)(x-\bar{x})\}}{(2\Pi)^{1/2} s_x} \ \exp\left(- \frac{(x-\bar{x})^2}{2s_x^{\ 2}}\right) dx$$

$$= \int_0 \frac{\bar{y}}{(2\Pi)^{1/2} s_x} \exp\left[-\frac{(x-\bar{x})^2}{2s_x^{\,2}}\right] dx +$$

$$\int_0 \frac{(rs_y/s_x)(x-\bar{x})}{(2\Pi)^{1/2} s_x} \exp\left[-\frac{(x-\bar{x})^2}{2s_x^{\,2}}\right] dx$$

$$= \bar{y}/2\ [1+ \operatorname{erf}(\bar{x}/\sqrt{2}s_x)] + [rs_y\ /(2\Pi)^{1/2}] \exp(-\bar{x}^2/2s_x^2)$$

$$= \bar{y}/2\ [1+ \operatorname{erf}(\bar{x}/\sqrt{2}s_x)] + [(R_{xy} - \bar{x}\bar{y})/\sqrt{2\Pi} s_x] \exp(-\bar{x}^2/2s_x^2) \quad \text{(B.3)}$$

Note that

$$r = \frac{R_{xy} - \bar{x}\bar{y}}{s_x s_y} \quad \text{(B.4)}$$

so that

$$R_{yU} = \frac{R_{xy} - \bar{x}\bar{y}}{(2\Pi)^{1/2} s_x} \exp\left[\frac{-\bar{x}^2}{2s_x^{\,2}}\right] + \frac{\bar{y}}{2}\left[1 + \operatorname{erf}\left(\frac{\bar{x}}{\sqrt{2}s_x}\right)\right] \quad \text{(B.5)}$$

Note that Nuttall has shown (4:18) that a result similar to Bussgang's can be obtained for any instantaneous non-linearity as long as the random process (noise) is separable. A random process is separable if

$$g(x_2, t_0) = \int (x_1 - x) p(x_1, x_2, t_0) dx_1$$

$$= g_1(x_2) g_2(t_0) \quad \text{(B.6)}$$

The joint Gaussian density is a separable process.

Appendix C. Data.

Explanation of headings:

t = Delay in milliseconds
$\overline{x}$ = Mean of the Gaussian noise
$\overline{x'}$ = Mean of the delayed noise
$\overline{y}$ = Mean of the LTI system (filter) output
$\overline{x'^2}$ = Mean square value of delayed noise
$\overline{y^2}$ = Mean square value of the filter output
R_{yU} = Crosscorrelation of the filter output and nonlinearity output
$R'_{yx'}$ = Calculated value of the crosscorrelation of the filter output and the delayed noise
$R_{yx'}$ = Crosscorrelation of the filter output and the delayed noise

Table 2. Filter "A" Data

t	$\overline{x}$	$\overline{x'}$	$\overline{y}$	$\overline{x'^2}$	$\overline{y^2}$	R_{yU}	$R'_{yx'}$	$R_{yx'}$
0	.01585	.01487	.01574	3.5154	.1026	.04934	.195	.194
0	-.00835	-.01366	-.00853	3.48	.0968	.03419	.180	.1812
0	.0088	-.00374	.0086	3.4332	.093	.04242	.177	.17915
2	-.0163	-.0126	-.01665	3.4704	.1004	.02803	.170	.1697
2	.0108	-.00604	.0106	3.4866	.0918	.0385	.176	.1552
2	-.00675	-.00822	-.00695	3.4788	.096	.03092	.161	.1601
4	-.0077	-.0004	-.00805	3.4758	.0956	.02603	.140	.14155
4	-.00685	-.01145	-.00706	3.4602	.0978	.0295	.154	.1439
4	.00395	.00008	.00369	3.4518	.0956	.0311	.136	.13775
6	.0005	.00166	.00025	3.4512	.0992	.02855	.132	.13115
6	-.00325	-.01326	-.00363	3.4722	.0936	.02457	.123	.1223
6	-.0069	-.01737	-.00704	3.4626	.0942	.0223	.120	.1233
8	.008	.00905	.00775	3.441	.0986	.02869	.115	.11765
8	-.00295	-.00242	-.0032	3.4992	.0982	.02174	.109	.11405
8	.00255	-.00506	.00236	3.4752	.097	.02495	.111	.1125
10	-.0056	-.01153	-.00571	3.48	.0942	.01812	.0981	.09905
10	-.0045	.00411	-.00475	3.447	.0964	.01944	.102	.0983
10	-.00375	-.0131	-.00381	3.4524	.0986	.01954	.0999	.1013

t	$\overline{x}$	$\overline{x'}$	$\overline{y}$	$\overline{x'^2}$	$\overline{y^2}$	R_{yU}	$R'_{yx'}$	$R_{yx'}$
12	-.00205	-.00064	-.00225	3.477	.0984	.01876	.0929	.0932
12	.0011	-.00168	.00092	3.438	.0936	.01833	.0831	.0846
12	-.0023	.00575	-.0025	3.4824	.1016	.01839	.0919	.0944
14	.009	-.00066	.00882	3.4458	.0964	.02024	.0737	.07695
14	-.0043	-.01025	-.00454	3.4014	.0936	.0134	.0724	.07445
14	-.0119	-.01377	-.01212	3.4794	.099	.01149	.0821	.07935
16	.0003	-.00016	.0013	3.4434	.0938	.01364	.0604	.0653
16	-.01065	-.01852	-.0107	3.5118	.0996	.01024	.0733	.07535
16	-.0097	-.00936	-.00995	3.4512	.096	.00833	.0620	.067
18	-.01645	-.02733	-.01667	3.4398	.1018	.00547	.0642	.0642
18	-.0116	-.01653	-.01184	3.4716	.098	.00539	.0528	.05815
18	-.00245	-.00516	-.00247	3.4734	.0988	.01135	.0588	.061
20	-.0097	-.00585	-.00986	3.4692	.0964	.00522	.0474	.04855
20	.0034	-.00323	.00309	3.4758	.0962	.01088	.0436	.0492
20	-.0022	-.00302	-.00252	3.4518	.094	.01087	.0565	.05365
22	-.00695	-.01764	-.0072	3.45	.0994	.00794	.0537	.053
22	-.00105	-.00384	-.00112	3.4224	.093	.00999	.0489	.0469
22	.01215	.00994	.01211	3.504	.1008	.01681	.0505	.05325
24	.0081	.0084	.00796	3.5178	.0976	.01327	.0437	.04495
24	-.0052	-.01541	-.00546	3.4338	.0928	.00524	.0370	.0375
24	.0014	.00609	.0014	3.4266	.0934	.00922	.0395	.0461
26	.0072	.00386	.00705	3.4752	.0972	.01125	.0361	.03685
26	-.0098	-.02247	-.00994	3.4986	.094	.00124	.0291	.0343
26	.005	.00917	.00494	3.4374	.0932	.00945	.0324	.0351
28	.01325	.01732	.01336	3.5034	.0988	.01213	.0256	.03185
28	-.01385	-.02735	-.01388	3.48	.0952	-.00003	.0324	.0312
28	-.00895	-.02478	-.00889	3.462	.0978	.00247	.0323	.0336
30	-.00175	-.0013	-.00179	3.456	.101	.00493	.0271	.0311
30	-.00195	.00551	-.00207	3.4458	.0954	.00438	.0252	.02875
30	.0077	-.00679	.00775	3.4818	.093	.00984	.0279	.02655
32	-.003	-.01883	-.00293	3.4884	.0978	.00297	.0208	.024
32	-.0056	-.00872	-.00555	3.5016	.1022	.00264	.0254	.02875
32	.01015	.00068	.01024	3.4686	.0988	.01041	.0247	.02665
34	-.0039	-.01349	-.00377	3.4866	.1	.00204	.0184	.02285
34	-.0027	-.00841	-.00265	3.474	.0938	.00295	.0200	.0187
34	.0018	.00751	.00198	3.453	.00994	.00614	.024	.025

t	$\overline{x}$	$\overline{x'}$	$\overline{y}$	$\overline{x'^2}$	$\overline{y^2}$	R_{yu}	$R'_{yx'}$	$R_{yx'}$
36	-.00135	-.00987	-.0015	3.4542	.1002	.00238	.0146	.0211
36	-.0009	-.0083	-.00092	3.4482	.099	.0036	.0189	.02205
36	-.01385	-.01462	-.01371	3.4218	.096	.00374	.0145	.01775
38	.0041	.00958	.00414	3.4512	.0972	.00435	.0106	.0131
38	-.0062	.00425	-.00635	3.4554	.1002	.00058	.0175	.01865
38	.0023	.00068	.00221	3.4182	.0974	.00347	.0110	.0147
40	-.0153	-.01431	-.01536	3.474	.1002	-.0059	.0083	.01455
40	-.00785	-.00386	-.00778	3.465	.096	-.00187	.0094	.01605
40	-.0068	-.0068	-.00675	3.4554	.097	-.00172	.00772	.01275

Table 3. Filter "B" Data

t	$\overline{x}$	$\overline{x'}$	$\overline{y}$	$\overline{x'^2}$	$\overline{y^2}$	R_{yu}	$R'_{yx'}$	$R_{yx'}$
0	.01595	.01447	.01608	3.4722	.8514	.01992	.0555	.0584
0	.00225	.00216	.002055	3.4674	.86	.01138	.0483	.0557
0	-.00445	-.01205	-.00496	3.4746	.8614	.00274	.0244	.03905
2	-.0079	-.00529	-.00795	3.4836	.9096	.01869	.106	.11
2	.0073	-.00007	.00722	3.4404	.9136	.02725	.110	.10955
2	.00225	-.00853	.00212	3.5202	.8466	.02572	.116	.12095
4	-.00135	-.00325	-.00148	3.459	.8224	.03269	.156	.172
4	-.0064	-.00722	-.00636	3.4542	.8794	.03291	.168	.16985
4	.0058	.00016	.00562	3.5196	.8482	.03886	.170	.1648
6	.00025	-.01099	.00031	3.4812	.8678	.04782	.223	.2265
6	-.0037	.00109	-.00338	3.4434	.8388	.04224	.204	.2194
6	-.00035	-.0109	-.00048	3.4164	.9172	.04507	.210	.2173
8	-.00355	-.01419	-.00353	3.4392	.8858	.05291	.254	.25605
8	-.0007	.0013	-.00061	3.4404	.8144	.05201	.243	.2453
8	.0061	.01459	.00612	3.42	.8954	.06052	.266	.262
10	.00595	-.0073	.00614	3.4794	.9444	.06661	.297	.2922
10	.01285	.0192	.01291	3.4854	.8558	.06778	.287	.28865
10	.00945	.00852	.0096	3.4746	.8748	.06036	.260	.26625
12	.0062	-.00579	.00627	3.4476	.8396	.06564	.291	.2832
12	-.00295	.00252	-.00266	3.4854	.8088	.05841	.280	.2846
12	.01315	.00914	.01317	3.438	.819	.0636	.265	.28105

t	$\overline{x}$	$\overline{x'}$	$\overline{y}$	$\overline{x'^2}$	$\overline{y^2}$	R_{yU}	$R'_{yx'}$	$R_{yx'}$
14	.01295	.00449	.01303	3.5076	.9218	.07315	.313	.3063
14	-.0057	-.01287	-.0056	3.4794	.8516	.05236	.258	.2734
14	.008	.0123	.00791	3.483	.9106	.06092	.266	.2926
16	.0031	-.0023	.00311	3.4248	.8894	.05811	.262	.2793
16	.00215	-.01459	.00202	3.4332	.8458	.0603	.275	.2699
16	.001	-.01793	.00093	3.4236	.8946	.05987	.276	.27475
18	-.0059	-.00111	-.00627	3.501	.8636	.05123	.255	.26135
18	.00865	-.00332	.00856	3.423	.878	.05558	.238	.25365
18	.00005	-.00259	-.00027	3.4428	.8964	.05171	.241	.23925
20	-.01365	-.01519	-.01406	3.5166	.8978	.04342	.237	.23715
20	-.0091	-.01114	-.00955	3.5034	.9392	.04201	.220	.2322
20	-.00795	-.00558	-.00837	3.435	.8968	.03942	.203	.21785
22	-.0008	-.00063	-.00107	3.474	.8836	.03354	.159	.1587
22	-.0162	.00323	-.01692	3.4734	.9182	.02575	.160	.16745
22	-.00715	-.00226	-.00772	3.4362	.8364	.0296	.155	.1509
24	-.0095	-.01138	-.00975	3.4782	.8762	.02351	.133	.1218
24	-.00475	-.00655	-.00542	3.4686	.9418	.02652	.136	.12815
24	-.0019	-.01242	-.00221	3.4818	.9084	.02252	.111	.1209
26	.00465	.01233	.00417	3.435	.8798	.01611	.0652	.06375
26	.0002	-.01244	-.00012	3.4872	.8618	.00966	.0455	.0602
26	-.0029	-.00015	-.00328	3.4974	.8718	.0124	.0658	.0706
28	-.0017	.00649	-.00206	3.4596	.8554	.00268	.0173	.03465
28	-.00295	-.00181	-.00349	3.4548	.9188	-.00096	.00366	.01425
28	-.00635	-.02183	-.00667	3.4632	.9618	.00503	.0390	.03865
30	.00195	.00916	.00154	3.4962	.8678	-.00777	-.0400	-.03965
30	.0015	.00671	.0012	3.4398	.9158	-.00901	-.0447	-.03835
30	-.00155	-.00457	-.00222	3.4536	.9006	-.00776	-.0310	-.02675
32	-.0007	.00034	.00094	3.4656	.8316	-.01956	-.0935	-.07135
32	-.01225	-.03594	-.0128	3.424	.8462	-.02303	-.0744	-.07025
32	-.00515	-.00286	-.00546	3.4782	.9256	-.02093	-.0851	-.0818
34	-.0075	-.01804	-.00786	3.423	.9008	-.03416	-.140	-.126
34	-.00815	-.01746	-.00852	3.4632	.8574	-.02989	-.120	-.12465
34	.0176	.0349	.01728	3.444	.8522	-.01893	-.128	-.12535
36	-.01245	-.02083	-.01269	3.44	.851	-.04015	-.157	-.1588
36	-.00535	-.00855	-.00541	3.4278	.8424	-.03719	-.160	-.15835
36	-.0045	.00079	-.00474	3.4488	.887	-.0329	-.142	-.13695

t	$\overline{x}$	$\overline{x^T}$	$\overline{y}$	$\overline{x'^2}$	$\overline{y^2}$	R_{yU}	$R'_{yx'}$	$R_{yx'}$
38	0	-.013	-.00042	3.486	.9516	-.04316	-.201	-.1958
38	-.00535	-.01182	-.00581	3.4506	.8998	-.04288	-.186	-.18755
38	-.00025	-.00797	-.00071	3.4866	.845	-.04107	-.191	-.17565
40	-.00825	-.01175	-.00874	3.4368	.8592	-.04412	-.185	-.18665
40	-.00525	-.00814	-.00555	3.4626	.8494	-.04809	-.211	-.1904
40	-.0074	-.01656	-.00774	3.4896	.8722	-.05059	-.219	-.2014
42	.0071	.00413	.00668	3.453	.8868	-.0445	-.223	-.2196
42	-.0103	-.01232	-.01066	3.447	.8894	-.05268	-.220	-.21425
42	.00315	.02144	.00259	3.4944	.904	-.04694	-.226	-.20825
44	.0073	.00615	.0068	3.4782	.8226	-.03817	-.194	-.19905
44	-.0045	-.02158	-.00491	3.4464	.8918	-.04985	-.221	-.20945
44	.00565	.00961	.00531	3.4674	.8894	-.04308	-.213	-.22005
46	.0101	.00184	.01032	3.4854	.8628	-.03558	-.191	-.1861
46	-.00075	.0063	-.00078	3.4896	.8468	-.04205	-.195	-.19615
46	.0057	.00579	.00525	3.4806	.8912	-.04242	-.211	-.20695
48	-.00275	.00683	-.00305	3.4788	.8424	-.04051	-.182	-.18465
48	-.00065	.00374	-.00111	3.4104	.8758	-.03945	-.180	-.17705
48	.00475	-.00186	.00422	3.4974	.8918	-.03834	-.190	-.1834
50	.0049	.00529	.00457	3.4932	.9506	-.04047	-.200	-.1886
50	.001	-.00312	.00057	3.432	.8582	-.03046	-.143	-.1456
50	-.01385	-.00819	-.01405	3.4596	.8906	-.04176	-.162	-.15645
52	-.01395	-.01243	-.01415	3.4476	.8494	-.03256	-.119	-.11875
52	-.0004	-.00977	-.00084	3.495	.8778	-.02861	-.132	-.12035
52	.00305	-.00025	.00271	3.4536	.8462	-.02644	-.129	-.12475
54	.0042	.00238	.00409	3.5088	.8366	-.01798	-.0940	-.089
54	-.00405	-.01816	-.00449	3.4728	.933	-.02522	-.107	-.09905
54	.0021	-.00839	.00204	3.4806	.8768	-.01745	-.0864	-.0916
56	.00395	.01281	.00356	3.471	.8584	-.0082	-.0466	-.04165
56	-.00605	-.0062	-.00634	3.4896	.9456	-.01718	-.0656	-.04835
56	.0052	.00062	.00472	3.45	.8558	-.00787	-.0476	-.04365
58	-.00035	-.0009	-.00067	3.45	.8912	-.00442	-.019	-.008
58	.00525	.00039	.00517	3.4446	.876	-.00326	-.0272	-.01615
58	-.00655	-.0112	-.007	3.489	.9114	-.00669	-.0149	-.01695
60	.00325	-.01509	.00331	3.465	.8524	.01117	.0444	.0372
60	.01625	.01753	.01598	3.4686	.819	.01289	.0229	.02445
60	-.0007	-.00388	-.00085	3.4602	.8508	.00449	.0229	.0206

t	$\overline{x}$	$\overline{x^T}$	$\overline{y}$	$\overline{x'^2}$	$\overline{y^2}$	R_{yU}	$R'_{yx'}$	$R_{yx'}$
62	.0007	-.0039	.00068	3.4746	.8096	.011	.0498	.05225
62	.02285	.01228	.02291	3.4842	.8656	.01937	.0371	.04985
62	-.0091	-.01879	-.00912	3.4722	.8936	.01068	.0712	.0667
64	.0076	.00978	.00749	3.4806	.9076	.02251	.0878	.09475
64	.00985	-.02771	-.00999	3.4404	.898	.00983	.0689	.08425
64	.0018	.00484	.00169	3.4416	.8634	.02005	.0893	.09185
66	-.00935	-.02031	-.00948	3.5034	.9042	.0206	.119	.12095
66	.0055	.00093	.00538	3.496	.7792	.0228	.094	.09635
66	.0041	.00211	.00423	3.5016	.8964	.02619	.113	.11815
68	-.0003	-.01319	-.00079	3.4848	.8758	.03062	.145	.14255
68	.0036	.00179	.00368	3.4506	.9152	.03033	.133	.1363
68	.00705	.00001	.00693	3.4548	.824	.02818	.115	.1189
70	-.0029	-.00192	-.00302	3.4854	.8958	.03228	.158	.1708
70	.0002	-.01426	.00021	3.5148	.8368	.03457	.162	.15525
70	.00155	.00387	.00177	3.4878	.8954	.0363	.166	.1625
72	.0066	-.005	.0066	3.4536	.8238	.03214	.134	.131
72	.00765	-.00186	.00742	3.4674	.7962	.03465	.1444	.1526
72	-.01005	-.02471	-.00978	3.5052	.9122	.03012	.164	.17495
74	.0063	-.00587	.00628	3.4584	.901	.04254	.184	.1721
74	.00145	-.01392	.00139	3.4614	.8356	.03493	.160	.158
74	.0007	.00315	.00038	3.4632	.8772	.03048	.141	.1521
76	.01245	.01893	.01258	3.4734	.8429	.03819	.149	.1519
76	.0006	.00449	.00032	3.4536	.8778	.03129	.145	.14955
76	-.0066	-.00252	-.00671	3.456	.8884	.03301	.169	.1627
78	-.00365	.00809	-.00407	3.4794	.921	.03099	.154	.1505
78	.00435	.01621	.00423	3.471	.9032	.03653	.161	.155
78	-.007	-.02418	-.00721	3.4764	.8934	.02696	.143	.1326
80	.0157	.01634	.01529	3.504	.8768	.03355	.122	.1301
80	-.0003	-.01726	-.00039	3.399	.8948	.02543	.118	.11615
80	.00905	.00583	.00925	3.468	.863	.02633	.102	.1111
82	-.0013	-.01126	-.00132	3.4872	.8936	.02599	.125	.12765
82	.00725	.00515	.00725	3.45	.8492	.02448	.0971	.1043
82	.0091	.00657	.00909	3.4908	.856	.02989	.119	.1038
84	-.00315	-.00121	-.00316	3.4494	.8568	.01556	.0798	.0759
84	-.0068	-.0248	-.00707	3.4686	.8666	.00692	.0488	.05665
84	-.0004	-.01302	-.00046	3.4704	.8506	.01208	.0766	.0602

t	$\overline{x}$	$\overline{x'}$	$\overline{y}$	$\overline{x'^2}$	$\overline{y^2}$	R_{yU}	$R'_{yx'}$	$R_{yx'}$
86	.0088	.003	.00891	3.45	.8796	.01458	.0471	.0513
86	.0056	.00138	.00548	3.4512	.8696	.00896	.0290	.0381
86	.00615	-.00552	.00623	3.4668	.8138	.01057	.0348	.0361

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This investigation applied the Bussgang theorem to the crosscorrelation method of linear, time invariant (LTI) system identification. In this procedure a Gaussian signal is passed through an LTI system and the output is crosscorrelated with a non-linearly distorted version of the original Gaussian signal. If the Gaussian noise were white the crosscorrelation would be equal to the impulse response of the LTI system within a constant of proportion ality. With the use of bandlimited Gaussian noise this relationship is only approximately satisfied.

The analysis compared the performance of the cross-correlation method with the non-linearity to that of the crosscorrelation method without the non-linearity. The experimental results indicate that the introduction of the non-linearity degrades the performance of the method, but the this can be improved by correcting for the effects of several quantities associated with the time varying statistics of the Gaussian noise.

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